Name (IN CAPITALS): **Version #1**

Instructor and section number or class time: Shaun The Sheep, (break of dawn)

**Math 10560 Exam 3**
**Apr. 23 2024**

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators are not allowed.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all 16 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

| | | | | | |
|---|---|---|---|---|---|
| 1. | (•) | (b) | (c) | (d) | (e) |
| 2. | (•) | (b) | (c) | (d) | (e) |
| 3. | (•) | (b) | (c) | (d) | (e) |
| 4. | (•) | (b) | (c) | (d) | (e) |
| 5. | (•) | (b) | (c) | (d) | (e) |
| 6. | (•) | (b) | (c) | (d) | (e) |
| 7. | (•) | (b) | (c) | (d) | (e) |
| 8. | (•) | (b) | (c) | (d) | (e) |
| 9. | (•) | (b) | (c) | (d) | (e) |
| 10. | (•) | (b) | (c) | (d) | (e) |

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|---|---|
| **Multiple Choice** | |
| 11. | |
| 12. | |
| 13. | |
| 14. | |
| Total | |

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| 2 | (a) | (b) | (c) | (d) | (e) |
| 3. | (a) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | (e) |
| 5. | (a) | (b) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (e) |
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| **Multiple Choice** | ________ |
| 11. | ________ |
| 12. | ________ |
| 13. | ________ |
| 14. | ________ |
| Total | ________ |

## Multiple Choice

**1.**(6pts) Consider the following series;

$$(I) \quad \sum_{n=1}^{\infty} \frac{2\sqrt{n}+1}{n^2+2n+1} \qquad (II) \quad \sum_{n=1}^{\infty} \frac{2^n}{n3^n} \qquad (III) \quad \sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$$

Use comparison tests to determine which of the following is **true**?

(I) Set $b_n = \frac{2\sqrt{n}+1}{n^2+2n+1}$ and $a_n = \frac{1}{n^{3/2}}$. Then

$$\lim_{n\to\infty} \frac{b_n}{a_n} = \lim_{n\to\infty} \frac{\frac{2\sqrt{n}+1}{n^2+2n+1}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{n^{3/2}(2\sqrt{n}+1)}{n^2+2n+1} = \lim_{n\to\infty} \frac{2n^2+n^{3/2}}{n^2+2n+1} = 2$$

where the final equality can be seen by dividing each term by $n^2$ and taking the limit of each term separately. Since this limit is positive, the limit comparison test says that $\sum b_n$ converges if $\sum a_n$ converges. Indeed,

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

converges by the $p$-test, where here $p = 3/2 > 1$.

(II) Observe that

$$\frac{2^n}{n3^n} < \frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n$$

where

$$\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$$

converges, being a geometric series with common ratio $r = 2/3 < 1$. Thus by the basic comparison test,

$$\frac{2\sqrt{n}+1}{n^2+2n+1}$$

converges.

(III) Observe that $e^{1/n} > 1$ or for all $n$. Thus

$$\frac{e^{1/n}}{n} > \frac{1}{n}$$

and as the harmonic series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

diverges, so too does $\sum \frac{e^{1/n}}{n}$ by basic comparison.

(a) (III) diverges while (I) and (II) converge.

(b) All three of the series converge.

(c) (II) diverges while (I) and (III) converge.

(d) All three of the series diverge.

(e) (I) and (II) diverge while (III) converges.

**2.**(6pts) Consider the following series;

$$(I)\quad \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2} \qquad (II)\quad \sum_{n=2}^{\infty}\frac{(-1)^n\sqrt{n}}{\ln(n)} \qquad (III)\quad \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n+1}}$$

Which of the following is **true**?

Recall that a series $\sum a_n$ converges absolutely if the series $\sum |a_n|$ converges.

(I) We have that

$$\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{n^2}\right| = \sum_{n=1}^{\infty}\frac{1}{n^2}$$

which converges by the $p$-test (here $p = 2 > 1$).

(II) We use the test for divergence to show that this series diverges. Recall that an alternating *sequence* $(-1)^n b_n$ (here $b_n \geq 0$) converges if and only if

$$\lim_{n\to\infty} b_n = 0.$$

In our case,

$$b_n = \frac{\sqrt{n}}{\ln(n)}$$

and one finds using L'Hopitals rule that

$$\lim_{n\to\infty}\frac{\sqrt{n}}{\ln(n)} = \lim_{n\to\infty}\frac{\frac{1}{2\sqrt{n}}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{\sqrt{n}}{2} = \infty.$$

As stated above we have shown that

$$\lim_{n\to\infty}\frac{(-1)^n\sqrt{n}}{\ln(n)}$$

does not exist, and so the series $\sum \frac{(-1)^n\sqrt{n}}{\ln(n)}$ diverges by the test for divergence.

(III) This series converges by the alternating series test: we need to show that

$$\lim_{n\to\infty}\frac{1}{\sqrt{n+1}} = 0$$

and that $a_n := 1/\sqrt{n+1}$ as $n$ increases. The fact that the limit is 0 is clear. To see that $a_{n+1} < a_n$, compute

$$n+2 > n+1 \implies \sqrt{n+2} > \sqrt{n+1} \implies a_{n+1} = \frac{1}{\sqrt{n+2}} < \frac{1}{\sqrt{n+1}} = a_n$$

(a) (I) is absolutely convergent (II) diverges and (III) is conditionally convergent.

(b) (I) is absolutely convergent while (II) and (III) are conditionally convergent.

(c) (I) and (III) are absolutely convergent while (II) is conditionally convergent.

(d) All three of the series are conditionally convergent.

(e) (I) is conditionally convergent while (II) and (III) are divergent.

**3.**(6pts) Determine which **one** of the following series is convergent.

The correct answer is (a): Set $b_n = \frac{n2^{1/n}}{n^3+1}$ and $a_n = \frac{1}{n^2}$ and compute the following limit:

$$\lim_{n\to\infty} \frac{b_n}{a_n} = \lim_{n\to\infty} \frac{n^3 2^{1/n}}{n^3+1} = \left(\lim_{n\to\infty} \frac{n^3}{n^3+1}\right)\left(\lim_{n\to\infty} 2^{1/n}\right) = 1.$$

Since this limit is positive and $\sum a_n = \sum \frac{1}{n^2}$ converges by the $p$-test (here $p = 2 > 1$), the limit comparison test says that $\sum b_n$ converges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n2^{1/n}}{n^3+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{3^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 5^n}{3^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1-\sin(1/n)}{n}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{2^n+n}$

**4.**(6pts) Consider the following series;

$$(I) \quad \sum_{n=1}^{\infty} \left( \frac{\sqrt{n}+1}{2n+1} \right)^n \qquad\qquad (II) \quad \sum_{n=2}^{\infty} \frac{(-1)^n 2^n \sqrt{n}}{(n-2)!}$$

Which of the following is **true**?

Series (I) converges by the root test:

$$\lim_{n\to\infty} |a_n|^{1/n} = \lim_{n\to\infty} \frac{\sqrt{n}+1}{2n+1} = 0 < 1.$$

Series (II) converges by the ratio test:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{2^{n+1}\sqrt{n+1}}{(n-1)!} \frac{(n-2)!}{2^n \sqrt{n}} = \lim_{n\to\infty} = \lim_{n\to\infty} \frac{2}{n-1} \sqrt{\frac{n+1}{n}} = 0 < 1.$$

(a) Both of the series converge.

(b) (I) diverges and (II) converges.

(c) Both of the series diverge.

(d) (I) converges and (II) diverges.

(e) The ratio test applied to (II) is inconclusive.

**5.**(6pts) Find a power series representation for the function

$$\frac{x}{1+x^3}$$

in the interval $(-1, 1)$.

$$\frac{x}{1+x^3} = x\left(\frac{1}{1-(-x^3)}\right) = x\sum_{n=0}^{\infty}(-x^3)^n = x\sum_{n=0}^{\infty}(-1)^n x^{3n} = \sum_{n=0}^{\infty}(-1)^n x^{3n+1}.$$

(a) $\sum_{n=0}^{\infty}(-1)^n x^{3n+1}$ (b) $\sum_{n=0}^{\infty}(-1)^n x^{3n+3}$ (c) $\sum_{n=0}^{\infty}(-1)^n x^{3n}$

(d) $\sum_{n=0}^{\infty} x^{3n+1}$ (e) $\sum_{n=0}^{\infty} x^{3n+3}$

**6.**(6pts) Consider the function $f(x)$ defined as

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n n!}, \quad -\infty < x < \infty.$$

Which of the following statements is true?

There are two approaches: one is to termwise antidifferentiate $f(x)$ to get a power series expansion for an antiderivative $F(x)$ of $f(x)$ such that $F(0) = 0$. Then

$$\int_0^1 f(x)dx = F(1)$$

where $F(1)$ will already be written as a series. The other approach (will be given in full here) is to recognize that $f(x) = e^{-x/3}$. To see this, write

$$e^{-x/3} = \sum_{n=0}^{\infty} \frac{(-x/3)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n n!} = f(x).$$

Now,

$$\int_0^1 f(x)dx = \int_0^1 e^{-x/3}dx = -3e^{-x/3}\Big|_0^1 = 3 - 3e^{-1/3}$$

Now we expand $e^{-1/3}$ using the power series for $e^x$:

$$3 - 3e^{-1/3} = 3 - 3\sum_{n=0}^{\infty} \frac{(-1/3)^n}{n!} = -3\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n!} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3^{n-1} n!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^n (n+1)!}$$

(a) $\displaystyle\int_0^1 f(x)dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^n (n+1)!}$

(b) $\displaystyle\int_0^1 f(x)dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^n n!}$

(c) $\displaystyle\int_0^1 f(x)dx = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{3^{n+1} (n+1)!}$

(d) $\displaystyle\int_0^1 f(x)dx = \left(\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n (n+1)!}\right) - 1$

(e) $\displaystyle\int_0^1 f(x)dx = \left(\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{3^n n!}\right) - 1$

**7.**(6pts) Which of the following gives a power series for $\sin\left(\frac{x^2}{2}\right)$?

Recall the power series expansion for $\sin\theta$ about $\theta = 0$:

$$\sin\theta = \sum_{n=0}^{\infty} \frac{(-1)^n \theta^{2n+1}}{(2n+1)!}.$$

We now set $\theta = x^2/2$ and compute:

$$\sin\left(\frac{x^2}{2}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^2/2)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{2^{2n+1}(2n+1)!}.$$

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{2^{2n+1}(2n+1)!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{2^{2n}(2n)!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{2^{2n+1}(2n+1)!}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{2^{2n}(n)!}$

**8.**(6pts) The degree 2 Taylor polynomial of

$$f(x) = \sqrt{x+1}$$

centered at $a = 1$ is given by:

We need to compute $f(1), f'(1)$ and $f''(1)$. Clearly $f(1) = \sqrt{2}$. Next,

$$f'(x) = \frac{1}{2\sqrt{(x+1)}}$$

so that

$$f'(1) = 1/2\sqrt{2}.$$

Finally,

$$f''(x) = \frac{d}{dx}\frac{1}{2\sqrt{x+1}} = \frac{-1}{4(x+1)^{3/2}}$$

so that

$$f''(1) = \frac{-1}{8\sqrt{2}}.$$

Combining these, we get

$$T_2(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 = \sqrt{x} + \frac{x-1}{2\sqrt{2}} - \frac{(x-1)^2}{16\sqrt{2}}$$

(a) $T_2(x) = \sqrt{2} + \dfrac{x-1}{2\sqrt{2}} - \dfrac{(x-1)^2}{16\sqrt{2}}$

(b) $T_2(x) = \sqrt{2} + \dfrac{x-1}{2\sqrt{2}} - \dfrac{(x-1)^2}{8\sqrt{2}}$

(c) $T_2(x) = \sqrt{2} + (x-1) - \dfrac{(x-1)^2}{2}$

(d) $T_2(x) = 1 + \dfrac{x}{2} - \dfrac{x^2}{8}$

(e) $T_2(x) = 1 + \dfrac{(x-1)}{2} - \dfrac{(x-1)^2}{8}$

**9.**(6pts) Use your knowledge of a well known power series to evaluate the following limit;

$$\lim_{x\to 0} \frac{e^{3x^5} - 1 - 3x^5}{x^{10}}.$$

$$\lim_{x\to 0} \frac{e^{3x^5} - 1 - 3x^5}{x^{10}} = \lim_{x\to 0} \frac{-1 - 3x^5 + e^{3x^5}}{x^{10}} = \lim_{x\to 0} \frac{1}{x^{10}} \left( -1 - 3x^5 + \sum_{n=0}^{\infty} \frac{3^n x^{5n}}{n!} \right) = \lim_{x\to 0} \frac{1}{x^{10}} \sum_{n=2}^{\infty} \frac{3^n x^{5n}}{n!}$$

where we have cancelled the terms $n = 0$ and $n = 1$ of the series with $-1 - 3x^5$. Continuing,

$$\lim_{x\to 0} \frac{e^{3x^5} - 1 - 3x^5}{x^{10}} = \lim_{x\to 0} \frac{1}{x^1 0} \sum_{n=2}^{\infty} \frac{3^n x^{5n}}{n!} = \lim_{x=0} \sum_{n=2}^{\infty} \frac{3^n x^{5n-10}}{n!}.$$

Now evaluating at $x = 0$, each term of the series is equal to zero except when $n = 2$, $x^{5n-10} = x^0 = 1$, so that

$$\lim_{x\to 0} \frac{e^{3x^5} - 1 - 3x^5}{x^{10}} = 3^2/2! = 9/2.$$

(a) $\frac{9}{2}$ (b) 9 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$ (e) 3

**10.**(6pts) Which of the following is a graph of the parametric curve defined by

$$x = \sin(t) + \frac{t}{\pi}, \qquad y = \cos(t) + \frac{t}{\pi},$$

for $0 \le t \le 3\pi$?

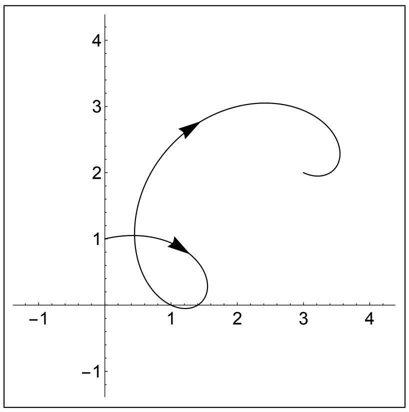
We check the endpoints $t = 0$ and $t = 3\pi$.

$$x(0) = \sin(0) + 0 = 0 \quad y(0) = \cos(0) + 0 = 1$$

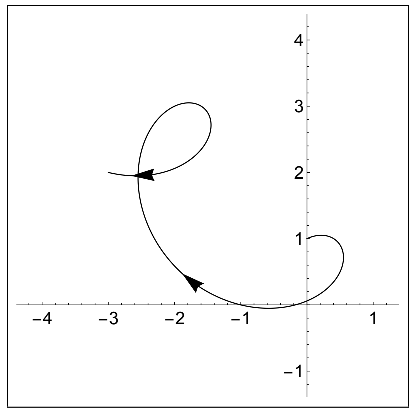
$$x(3\pi) = \sin(3\pi) + \frac{3\pi}{\pi} = 0 + 3 = 3 \quad y(3\pi) = \cos(3\pi) + \frac{3\pi}{\pi} = -1 + 3 = 2.$$

The only option which has these end points is (a).

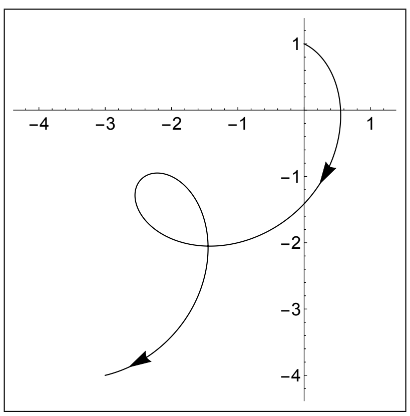
(a)



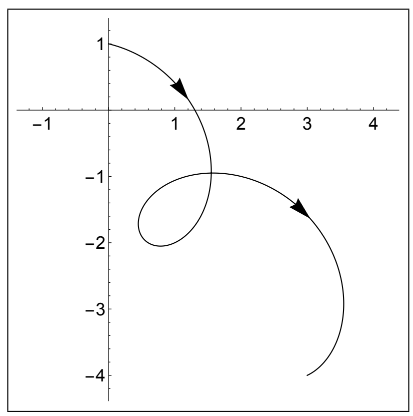

(b)



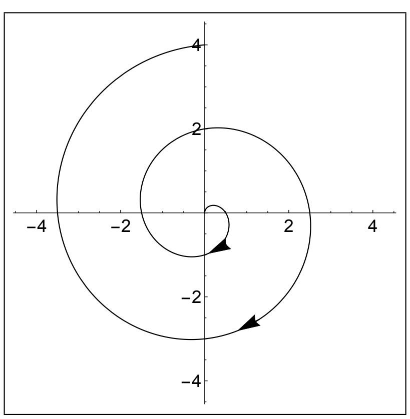

(c)




(d)




(e)



**Partial Credit**

Please show all of your work for credit in questions 11-13.

**11.**(13pts) Consider the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(n+1)}{n^2-n-4}$. Fill in the following blanks and in each case justify your answer. If you use a test of convergence, indicate which test you are using and show how it is applied (make sure you verify all of the details).

(a) Is the above series absolutely convergent? (**YES or NO**) NO

We must show that $\sum \frac{n+1}{n^2-n-4}$ is divergent. To do so we use the limit comparison test with

$$a_n = \frac{1}{n} \quad b_n = \frac{n+1}{n^2-n-4}.$$

We have

$$\lim_{n\to\infty} \frac{b_n}{a_n} = \lim_{n\to\infty} \frac{n^2+n}{n^2-n-4} = 1,$$

and so as this limit is positive and $\sum a_n = \sum \frac{1}{n}$ is divergent, the limit comparison test says that $\sum b_n$ is divergent as well.

(b) Is the above series conditionally convergent? (**YES or NO**) YES

We shall use the alternating series test. To do so we must show that

$$\frac{n+1}{n^2-n-4}$$

decreases as $n$ increases, and that

$$\lim_{n\to\infty} \frac{n+1}{n^2-n-4} = 0.$$

The limit is zero by a single application of L'Hopital's rule. Now, consider

$$f(x) = \frac{x+1}{x^2-x-4}.$$

To show that $f$ is a decreasing function, we can show that $f'(x) < 0$ for all $x \geq 0$:

$$f'(x) = \frac{(x^2-x-4)-(x+1)(2x-1)}{(x^2-x-4)^2} = \frac{-(x^2+2x+3)}{(x^2-x-4)^2}$$

This is negative if and only if

$$-(x^2+2x+3) < 0 \iff x^2+2x+3 > 0$$

which is certainly true if $x \geq 0$. Therefore $f(x)$ is decreasing for $x \geq 0$ and it follows that

$$f(n) = \frac{n+1}{n^2-n-4}$$

is decreasing.

**12.**(13pts) Find the radius of convergence and interval of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{3^n(n+1)}.$$

If, in the course of the solution, you test for convergence of a series, please state clearly which test you are using.

For the purposes of the ratio test we consider the limit

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{3^{n+1}(n+2)}\frac{3^n(n+1)}{(x-2)^n}\right| = \lim_{n\to\infty}\frac{|x-2|}{3}\frac{n+1}{n+2} = \frac{|x-2|}{3}.$$

The ratio test says that the series will converge when this limit is *strictly* less than 1, i.e. when $|x-2| < 3$, and that the series will diverge when this limit is *strictly* greater than 1, i.e. when $|x-2| > 3$. Thus the radius of convergence is 3. The open interval centered at 2 with radius 3 is $(-1, 5)$. For the full intercval of convergence, we must test the endpoints $x = -1$ and $x = 5$ with some method other than the ratio test.

For $x = -1$ we get the series

$$\sum_{n=1}^{\infty}\frac{(-1-2)^n}{3^n(n+1)} = \sum_{n=1}^{\infty}\frac{(-1)^n 3^n n}{3^n(n+1)} = \sum_{n=1}^{\infty}\frac{(-1)^n}{(n+1)}$$

which converges by the alternating series test.

For $x = 5$ we get the series

$$\sum_{n=1}^{\infty}\frac{5-2)^n}{3^n(n+1)} = \sum_{n=1}^{\infty}\frac{1}{n+1} = \sum_{n=2}^{\infty}\frac{1}{n}$$

which diverges by the $p$-test (here $p = 1 \geq 1$). Thus the interval of convergence is $[-1, 5)$.

**Radius Of Convergence:** $\underline{3}$ **Interval Of Convergence:** $\underline{[-1, 5)}$

**13.**(13pts) Consider the function $f(x)$ defined as $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{n+1} n!}$, for $-\infty < x < \infty$.

(a) Write down a power series expansion for the function $f'(x)$ on the interval $(-\infty, \infty)$.

$$f'(x) = \frac{d}{dx} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{n+1} n!} = \sum_{n=0}^{\infty} \frac{d}{dx} \frac{(-1)^n x^{2n}}{2^{n+1} n!} = \sum_{n=1}^{\infty} \frac{(-1)^n (2n x^{2n-1})}{2^{n+1} n!} = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{2^n (n-1)!}$$

(b) Write $f'(1)$ as the sum of an alternating series.

$$f'(1) = \sum_{n=1}^{\infty} \frac{(-1)^n}{2^n (n-1)!}$$

(c) Use the alternating series estimation theorem to estimate the value of $f'(1)$ so that the error of estimation is less than $\frac{1}{10}$.

The Alternating Series Estimation Theorem says that given a convergent alternating series

$$S = \sum_{n=1}^{\infty} (-1)^n b_n \quad \text{with } N\text{-th partial sum} \quad S_N = \sum_{n=1}^{N} (-1)^n b_n$$

we have the approximation $|S - S_N| < b_{N+1}$.

Now, we want to estimate $f'(1)$ by some $S_N$ with error less than $\frac{1}{10}$, which is to say

$$|f'(1) - S_N| < \frac{1}{10},$$

so we seek an $N$ such that $\frac{1}{10} > b_{N+1}$:

$$\frac{1}{10} > b_{N+1} = \frac{1}{2^{N+1} N!} \iff 2^{N+1} N! > 10.$$

One can check that $N = 1$ is too small but $N = 2$ satisfies the above inequality. By the Alternating Series Estimation Theorem, we can now take our estimate to be $S_2$, the second partial sum of

$$S = f'(1) = \sum_{n=1}^{\infty} \frac{(-1)^n}{2^n (n-1)!}.$$

That is,

$$\sum_{n=1}^{2} \frac{(-1)^n}{2^n (n-1)!} = \frac{-1}{2} + \frac{1}{4} = -\frac{1}{4}$$

(d) Is $f(x)$ increasing or decreasing at $x = 1$? Justify your answer.

By part (c) we have that

$$\left| f'(1) - \left(-\frac{1}{4}\right) \right| < \frac{1}{10} \implies f'(1) < -\frac{1}{4} + \frac{1}{10} = \frac{-10+4}{40} = \frac{-3}{20} < 0.$$

Since $f'(1) < 0$ it follows that $f$ is decreasing at $x = 1$

**14.**(1pts) You will be awarded this point if you write your section number or class time next to the name of your instructor and you mark your answers on the front page with an X (not an O) . You may also use this page for

**ROUGH WORK**

 Initials: ____________

**The following is the list of useful trigonometric formulas:**

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x-y) + \sin(x+y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x-y) - \cos(x+y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x-y) + \cos(x+y)\big)$$

$$\int \sec\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta = \ln|\csc\theta - \cot\theta| + C$$